

Combining life cycle assessment and economic input-output is based on the work of Wassily Leontief in the 1930s. Leontief developed the idea of input-output models of the U.S. economy and theorized about expanding them with non-economic data. But the computational power at the time limited uses of the Economic Input-Output method that required matrix algebra.

From the Input-Output accounts a matrix or table **A** is created that represents the direct requirements of the intersectoral relationships. The rows of **A** indicate the amount of output from industry *i* required to produce one dollar of output from industry *j*. These are considered the direct requirements – the output from first tier of suppliers directly to the industry of interest.

Next, consider a vector of final demand, **y**, of goods in the economy. The sector in consideration must produce **I**×**y** units of output to meet this demand. At the same time **A**×**y** units of output are produced in all other sectors. So, the result is more than demand for the initial sector, but also demand for its direct supplier sectors. The resulting total output,  $x_{\text{direct}}$  of the entire economy can be written

$$x_{\text{direct}} = (\mathbf{I} + \mathbf{A})\mathbf{y}$$

This relationship takes into account only one level of suppliers, however. The demand of output from the first-tier of suppliers creates a demand for output from *their* direct suppliers (i.e., the second-tier suppliers of the sector in consideration). For example, the demand for computers from the computer manufacturing sector results in a demand for semiconductors from the semiconductor manufacturing sector (first-tier). That in turn results in a demand from the electricity generation sector (second-tier) to operate the semiconductor manufacturing facilities. This demand continues throughout the economy. The output demanded from these second-tier sectors and beyond is considered indirect output.

The second-tier supplier requirements are calculated by further multiplication of the direct requirements matrix by the final demand, or **A**×**A**×**y**. In many cases, third and fourth or more tiers of suppliers exist, resulting in a summation of many of these factors so that the total output can be calculated as:

$$\mathbf{X} = (\mathbf{I} + \mathbf{A} + \mathbf{AA} + \mathbf{AAA} + \dots)\mathbf{y}$$

where **X** (with no subscript) is a vector including all supplier outputs, direct and indirect.

The expression **(I + A + AA + AAA + ...)** can be shown to be equivalent to **(I-A)<sup>-1</sup>**, which is called the total requirements matrix or the Leontief inverse. The relationship between final demand and total output can be expressed compactly as:

$$\mathbf{X} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{y} \text{ or } \Delta \mathbf{X} = (\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{y}$$

where the latter expression indicates that the EIO framework can be used to determine relative changes in total output based on an incremental change in final demand. Typically, the values in the matrices and vectors are expressed in dollar figures (i.e., in

the direct requirements matrix,  $\mathbf{A}$ , the dollar value of output from industry  $i$  used to produce one dollar of output from industry  $j$ ). This puts all items in the economy, petroleum or coal or electricity, into comparable units.

The economic input-output analysis can then be augmented with additional, non-economic data. One can determine the total external outputs associated with each dollar of economic output by adding external information to the EIO framework. First, the total external output per dollar of output is calculated from:

$$R_i = \text{total external output} / X_i$$

where  $R_i$  is used to denote the impact in sector  $i$ , and  $X_i$  is the total dollar output for sector  $i$ .

To determine the total (direct plus indirect) impact throughout the economy, the direct impact value is used with the EIO model. A vector of the total external outputs,  $B_i$ , can be obtained by multiplying the total economic output at each stage by the impact:

$$\Delta \mathbf{b}_i = \mathbf{R}_i \Delta \mathbf{X} = \mathbf{R}_i (\mathbf{I} - \mathbf{A})^{-1} \Delta \mathbf{y}$$

where  $\mathbf{R}_i$  is a matrix with the elements of the vector  $R_i$  along the diagonal and zeros elsewhere, and  $\mathbf{X}$  is the vector of relative change in total output based on an incremental change in final demand. A variety of impacts can be included in the calculation – resource inputs such as